POLITECNICO DI MILANO

Dipartimento di
Elettronica, Informazione
e Bioingegneria

Format Transforming Encryption:

More than Meets the DPI

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Credits & Outline

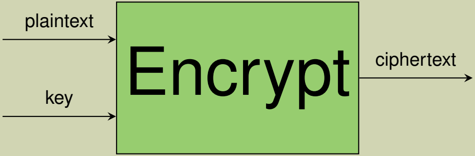
This talk is mainly based on the seminars of Tom Shrimtpon (Florida Institute for Cybersecurity Research) at Real World Crypto and Privacy summer school (Sibenik 5-9th June 2017)

Outline

1. FTE Definition
2. Ranking/unranking algorithms
3. Relaxed ranking
4. Build FTE framework
5. Application Scenarios: In-Place Encryption and Great Firewall of China
6. Further developments

Traditional Encryption vs FTE

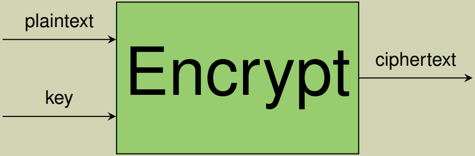
Traditional Encryption



Plaintexts and ciphertexts are bit strings!

Traditional Encryption vs FTE

Traditional Encryption



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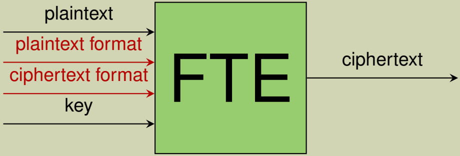
Format Transforming Encryption



- Ciphertexts must abide the specified format!
- Plaintexts and ciphertexts formats can be changed online

Format Definition

How Do We Define a Format?

- A format is a set of rules which defines the structure of the plain/ciphertexts
- Set of rules $\rightarrow$ a grammar which defines the syntax of the messages
- That is, a format basically defines a language
- A rather intuitive and user friendly way of specifying a language is a regular expression

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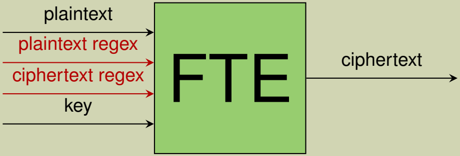
Format Transforming Encryption



Building FTE

2 approaches:

1. We use a bijection to map words of the language to a domain which is suitable for encryption algorithms (bit strings)

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Ranking

- An invertible, bijective mapping between words of a language and integers, according to an established lexicographical ordering
- Goldberg and Sipser proposed an algorithm to efficiently rank and unrank regular languages, starting from their Deterministic Finite Automaton (DFA) in 1985

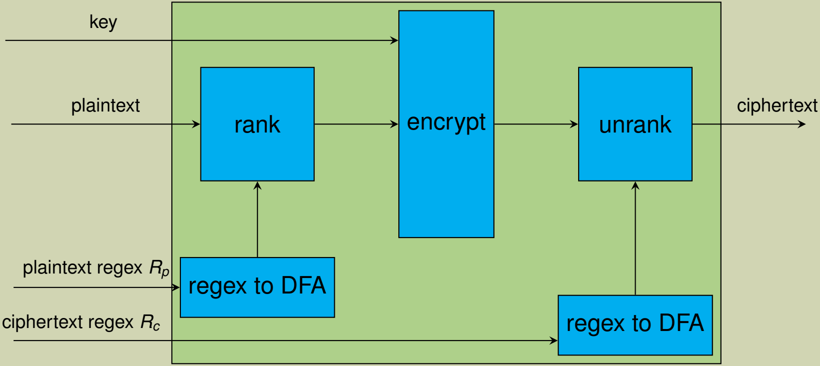
Formal definition:

$$rank : L \mapsto \mathbb{Z}_{|L|}, \qquad unrank : \mathbb{Z}_{|L|} \mapsto L \quad s.t. \quad \forall l \in L, \quad unrank(rank(l)) = l$$

Once we have a map to $\mathbb{Z}_{|L|}$, we can use whatever cipher to encrypt the plaintext!

Ranking Based FTE

Inside the FTE Box



- This construction is really flexible: change the regex to change the format
- Not all the formats can be used: $|L(R_p)| \leq |L(R_c)|$. Why?

Ranking Based FTE

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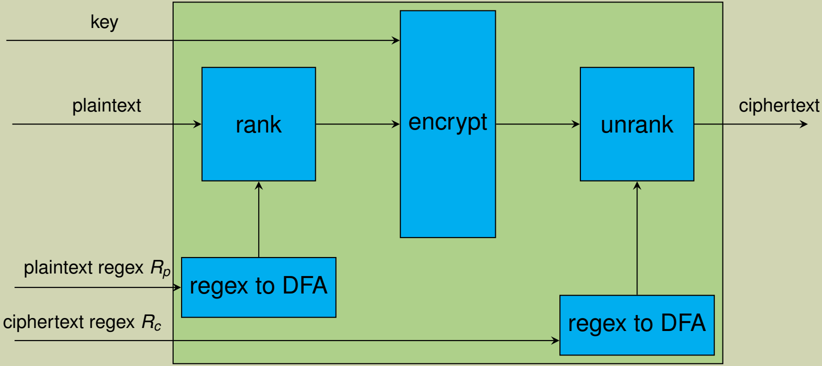

- This construction is really flexible: change the regex to change the format
- Not all the formats can be used: $|L(R_p)| \leq |L(R_c)|$. Why?
- Multiple plaintexts would map to the same ciphertext!

Ranking and Unranking

The basic brick of the framework: How to efficiently rank/unrank?

Ranking Idea

- Ranking requires a DFA representation of the language
- An ordering on the alphabet of the language is defined
- The ranking procedure proposed by Goldberg and Sipser can be conceptually split in 2 parts:
  1. The string $x \in L$ is mapped to its corresponding accepting path in the DFA (parsing)
  2. Accepting paths are ranked according to a lexicographical order
- A precomputed table of size $|\mathbb{Q}| \cdot max_{x \in L}(|X|)$ is employed
- With this precomputed table, ranking and unranking are $O(|X|)$

Precomputation

DFA Notation

A deterministic finite automaton (DFA) is a 5-tuple = $(\mathbb{Q}, \Sigma, \delta, q_0, \mathbb{F})$, where:

- $\mathbb{Q}$ is the set of states
- Σ is the alphabet of the language described by the DFA
- $\delta : \mathbb{Q} \times \Sigma \mapsto \mathbb{Q}$ is the transition function
- q_0 is the initial state
- $\mathbb{F}$ is the set of final states

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- Rank and unrank procedures employ the images of function $T : \mathbb{Q} \times \mathbb{N} \mapsto \mathbb{N}$
- $T(q, n)$ is the number of accepting paths starting from state q of length n
- Thus, n can be at most $max_{x \in L}(|x|)$
- This function can be precomputed and stored in tabular form

Precomputation

The function can be computed in a recursive fashion:

$$T(q,n) = \begin{cases} 1 & n = 0 \wedge q \in \mathbb{F} \\ 0 & n = 0 \wedge q \notin \mathbb{F} \\ \sum_a T(\delta(q,a), n-1) & \text{otherwise} \end{cases}$$

Precomputation

The function can be computed in a recursive fashion:

$$T(q,n) = \begin{cases} 1 & n = 0 \land q \in \mathbb{F} \\ 0 & n = 0 \land q \notin \mathbb{F} \\ \sum_a T(\delta(q,a), n-1) & \text{otherwise} \end{cases}$$

We can write a simple procedure to compute this function:

Table Precomputation

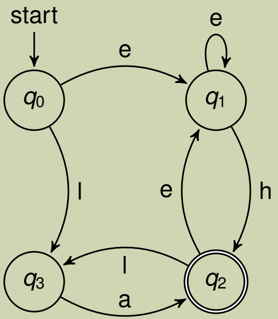
BUILD-TABLE(n)

for $q \in \mathbb{Q}$
  do if $q \in F$
    then $T(q,0) \leftarrow 1$
for $i \leftarrow 1$ **to** n
  do for $q \in \mathbb{Q}$
    do for $a \in \Sigma$
      do $T(q,i) \xleftarrow{+} T(\delta(q,a), i-1)$

Precomputation Example

Example format: $L = \{(e^{+}h|la)^{+}\}$

DFA for L



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Tabulated Function $T(q, n)$

States	n				
	0	1	2	3	4
q_0					
q_1					
q_2					
q_3					

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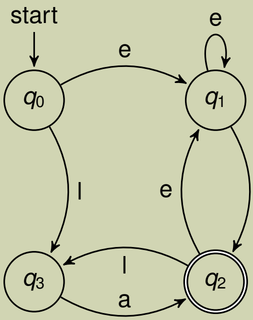

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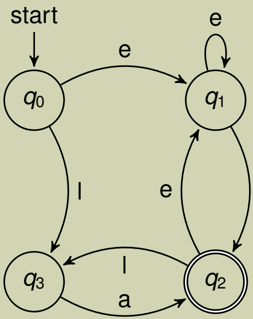

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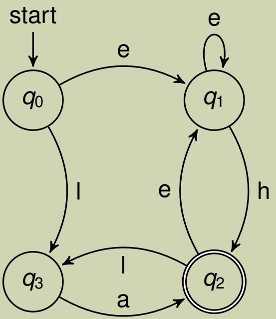

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Tabulated Function $T(q, n)$

States	n = 0	1	2	3	4
q_0	0	0	2		
q_1	0	1	1		
q_2	1	0	2		
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Precomputation Example

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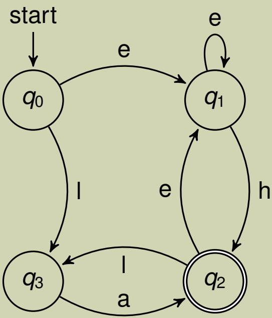

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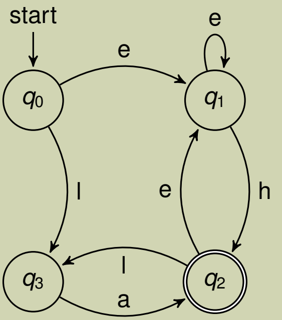

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States	n = 0	1	2	3	4
q_0	0	0	2	1	5
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Rank Algorithm

Exploiting $T(q, n)$ for Ranking

To rank a string $x \in L$:

- All the strings of the language whose length is less than $|x|$ can be easily counted by $\sum_{i=0}^{|x|-1} T(q_0, i)$
- We need an algorithm to compute the strings whose length is $|x|$ which are lexicographically less than x

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Again, rank can be recursively computed using $T(q, n)$:

$$rank(x[0 \dots n]) = \begin{cases} 0 & x = \epsilon \\ rank(x[1 \dots n]) + \sum_{\substack{a \in \Sigma \\ a < x[0]}} T(\delta(q, a), n-1) & \text{otherwise} \end{cases}$$

with q being updated according to the parsing of $x \rightarrow q = \delta(q, x[0])$ at each step

Rank Algorithm

Rank Algorithm

RANK(x)

$q \leftarrow q_0$
$rank \leftarrow 0$
for $i \leftarrow 0$ **to** $|x| - 1$
    do for $a \in \Sigma$
        do if $a \geq x[i]$
            then continue
          $rank \stackrel{+}{\leftarrow} T(\delta(q, a), |x| - i - 1)$
      $q \leftarrow \delta(q, x[i])$
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            rank +← T(δ(q, a), |x| − i − 1)
    q ← δ(q, x[i])
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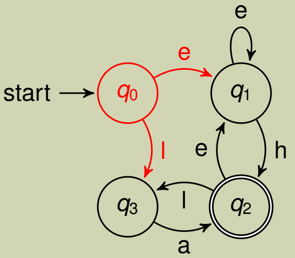
```

Example for $x = ehla$:

- $i = 0$
- $x[i] = e$
- $rank = 0$

DFA for L



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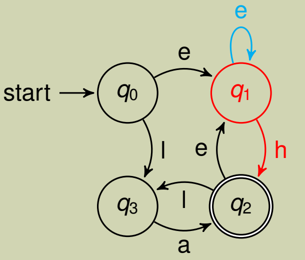
Example for $x = ehla$:

- $i = 1$
- $x[i] = h$
- $rank = 1$

DFA for L



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  $q \leftarrow \delta(q, x[i])$
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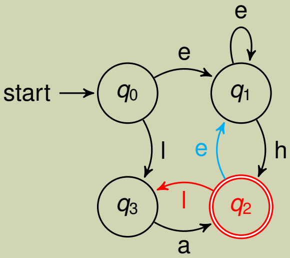
Example for $x = ehla$:

- $i = 2$
- $x[i] = l$
- $rank = 4$

DFA for L



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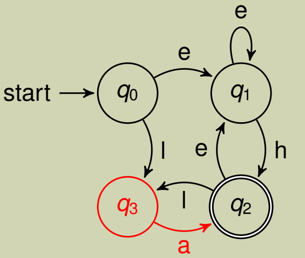

Example for $x = ehla$:

- $i = 3$
- $x[i] = a$
- $rank = 5$

Tabulated Function $T(q, n)$

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            do if a ≥ x[i]
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                rank +← T(δ(q, a), |x| − i − 1)
         q ← δ(q, x[i])
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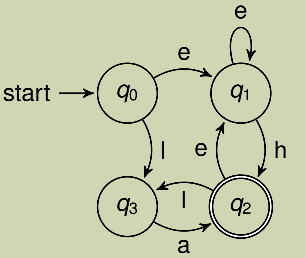
```

Example for $x = ehla$:

$rank = 5 \rightarrow$ There are 5 strings less than x: *eh*, *la*, *eeh*, *eeeh*, *eheh*

DFA for L



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Unrank Algorithm

Unranking Idea: We can get back the string due to the ”structure” of the rank

Rank Structure

- The rank of x is the number of strings lexicographically less than x
- Thus, we can consider the rank as a sum of all these strings
- We can partition the elements of this sum in order to use them to build x one character at a time

Unrank Algorithm

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- The rank of x is the number of strings lexicographically less than x
- Thus, we can consider the rank as a sum of all these strings
- We can partition the elements of this sum in order to use them to build x one character at a time
- Rank partitioning (given $n = |x|$):

$$
\begin{aligned}
r = {} & |\{s : |s| = 0\}| + |\{s : |s| = 1\}| + \cdots + |\{s : |s| = n-1\}| \\
& + |\{s : |s| = n \wedge s[0] < x[0]\}| + |\{s : |s| = n \wedge s[1] < x[1] \wedge s[0] = x[0]\}| \\
& + |\{s : |s| = n \wedge s[2] < x[2] \wedge s[0 \ldots 1] = x[0 \ldots 1]\}| + \cdots + \\
& + |\{s : |s| = n \wedge s[n-1] < x[n-1] \wedge s[0 \ldots n-2] = x[0 \ldots n-2]\}|
\end{aligned}
$$

- The size of these sets can be computed using $T(q, n)$ table, and used together with r to build x

Unrank Algorithm: Exploiting Partitions

Find $\mathbf{n} = |\mathbf{x}|$

- $|\{s : |s| = 0\}| = T(q_0, 0) \wedge |\{s : |s| = 1\}| = t(q_0, 1) \ldots$
- We sum up all $T(q_0, i)$, starting from $i = 0$, until when $sum > r$
- The i when we stop is equal to n

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Find $x[0]$

- Compute $r = r - \sum_{i=0}^{n-1} T(q_0, i)$
- $|\{s : |s| = n \wedge s[0] < x[0]\}| = \sum_{\substack{a \in \Sigma \\ a < x[0]}} T(\delta(q_0, a), n - 1)$
- We sum up all $T(\delta(q_0, a), n - 1)$, following alphabetical order to select the character a, until when $sum > r$
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- We sum up all $T(\delta(q_0, a), n-1)$, following alphabetical order to select the character a, until when $sum > r$
- The a when we stop is $x[0]$

Finding next character

- Compute $r = r - \sum_{\substack{a \in \Sigma \\ a < x[0]}} T(\delta(q_0, a), n-1)$
- Repeat the process used for $x[0]$ replacing $T(\delta(q', a), n-2)$, where $q' = \delta(q_0, x[0])$, with $T(\delta(q_0, a), n-1)$ in the summation

Unrank Algorithm

Code

UNRANK(r)

```
q ← q_0
n ← 0
while r ≥ T(q_0, n)
      do r ←(−) T(q_0, n)
         n ←(+) 1
x ← ε
for i ← 0 to n − 1
      do j ← 1
         while r ≥ T(δ(q, a_j), n − i − 1)
               do r ←(−) T(δ(q, a_j), n − i − 1)
                  j ←(+) 1
         x[i] ← a_j
         q ← δ(q, x[i])
```

Unrank Algorithm

Code

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$n \leftarrow 0$
while $r \geq T(q_0, n)$
    do $r \xleftarrow{-} T(q_0, n)$
        $n \xleftarrow{+} 1$
$x \leftarrow \epsilon$
for $i \leftarrow 0$ **to** $n - 1$
    do $j \leftarrow 1$
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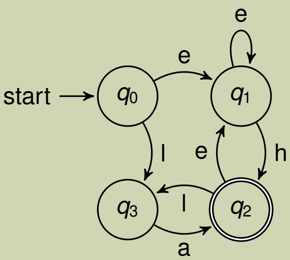
DFA for L



Example: unranking $r = 5$ for $L = (eh^+|la)^+$

- $r = 5$
- $n = 0$
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Tabulated Function $T(q, n)$

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Unrank Algorithm

Code

UNRANK(r)

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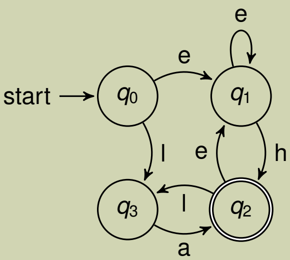

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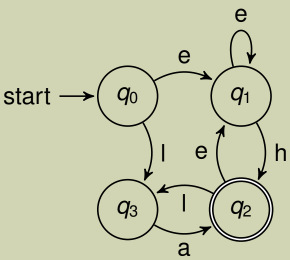
DFA for L



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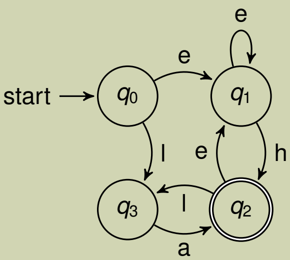
Example: unranking $r = 5$ for $L = (eh^+|la)^+$

- $r = 3$
- $n = 2$
- $x = \epsilon$

DFA for L



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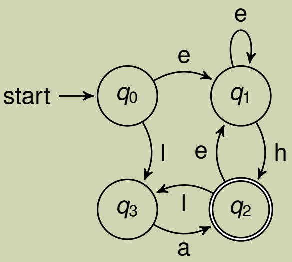
DFA for L



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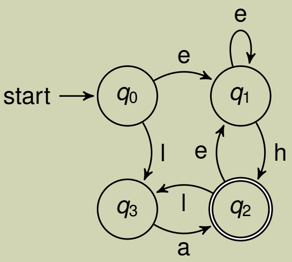

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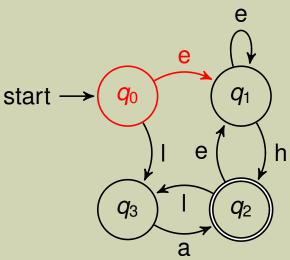

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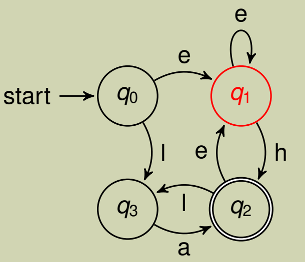
- $x = e$

DFA for L



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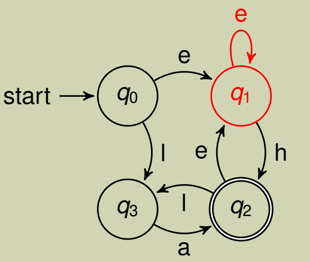
DFA for L



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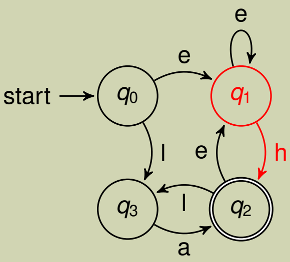
DFA for L



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Unrank Algorithm

Code

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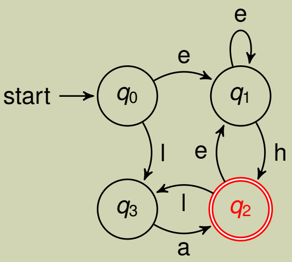
```

Example: unranking $r = 5$ for
$L = (eh^{+}|la)^{+}$

- $r = 1$
- $n = 4$
- $x =$ *eh*

DFA for L



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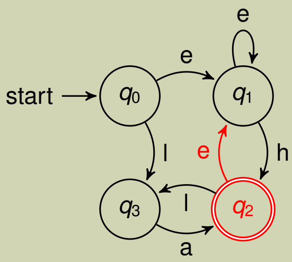
DFA for L



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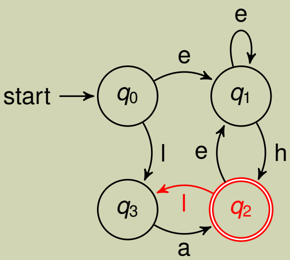
DFA for L



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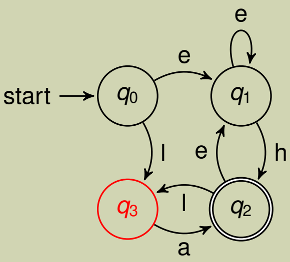
DFA for L



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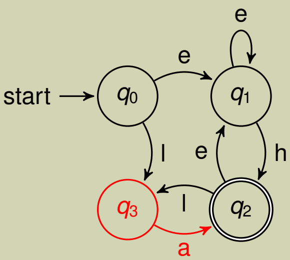

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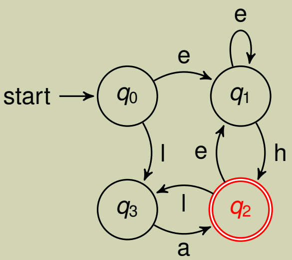
- $n = 4$
- $x =$ *ehla*

DFA for L



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From Regex To DFA

- Plaintext and ciphertext formats are regular expressions
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- For some languages, determinization of the NDFA leads to an exponential blowup of the number of states $|\mathbb{Q}|$ of the DFA
- To perform rank/unrank we need to precompute a table which is proportional to $|\mathbb{Q}|$

For some languages, ranking is extremely expensive in terms of memory consumption!

Relaxed Ranking

What if the ranking is performed from the N-DFA representation?

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Relaxed Ranking

We can still achieve our target by relaxing the ranking definition:

Formal Definition:

- $Rank : L \mapsto \mathbb{Z}_i, i > |L|$ is injective
- $Unrank : \mathbb{Z}_i \mapsto L$ is surjective
- $\forall x \in L \quad Unrank(Rank(x)) = x$

Ranking is a particular case of relaxed ranking where $i = |L|$, thus *rank* and *unrank* are bijections

Relaxed Ranking Design

Relaxed Ranking Idea

As with ranking, we can conceive the relaxed ranking as a 2 steps process

1. Map each string in the language to an accepting path in the corresponding N-DFA (parsing)
2. Rank (not relaxed) the set of all accepting paths

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- Since there is a bijection between the set of accepting paths $\mathcal{P}$ and $\mathbb{Z}_i \to i = |\mathcal{P}|$
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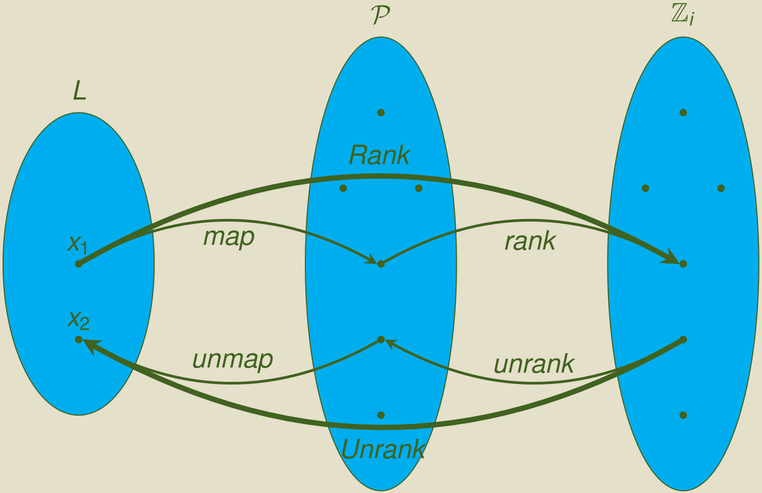
- In N-DFA, more than one accepting path for each string
- Therefore, we need to map a string to one of its accepting paths

Map function

The relaxed ranking requires 2 functions

- $map : L \mapsto \mathcal{P}$, which is injective
- $unmap : \mathcal{P} \mapsto L$, which is surjective
- Moreover, $\forall x \in L \quad unmap(map(x)) = x$

Relaxed Ranking Overview



Map Function

map $\equiv$ deterministically parse the string $\rightarrow$ possibly exponential time!

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Linear Time Parsing

Idea: 2 phases parsing

1. Build a graph which represents all the accepting paths for a string $x \in L$
2. Traverse this graph by choosing the lexicographically least one (fixing an order for the set $\mathbb{Q}$)

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Building the Graph

- In a N-DFA, the transition function is defined as $\delta : \mathbb{Q} \times \Sigma \mapsto \mathcal{P}(\mathbb{Q})$
- When we parse the k-th character of string x, we define a frontier $\mathcal{F}_k \subseteq \mathbb{Q} = \bigcup_{q \in \mathcal{F}_{k-1}} \delta(q, x[k-1])$, with the base case $\mathcal{F}_0 = \{q_0\}$
- $x \in L \Leftrightarrow \mathcal{F}_{|x|} \cap \mathbb{F} \neq \emptyset$
- The frontiers $\mathcal{F}_k$ contains all the possible states reachable from the initial one after parsing of k characters of x

But we need only the states belonging to accepting paths!

Map Function: Graph Building

Pruning Non-Accepting Paths

- We can enrich the information provided by the frontiers with backward frontiers
- A backward frontier $\mathcal{B}_k \subseteq \mathbb{Q} = \{q \mid \exists q' \in \mathcal{B}_{k+1}(\delta(q, x[k]) = q')\}$, with the base case $\mathcal{B}_{|x|} = \mathbb{F}$
- A frontier $\mathcal{B}_k$ represents the states which belongs to an accepting path for the substring $x[k \dots n-1]$, starting from any state of $\mathcal{B}_k$
- Now, define the frontier $\mathcal{S}_k = \mathcal{F}_k \cap \mathcal{B}_k \rightarrow$ these are the states which are reachable from the initial one after parsing the first k characters of x which leads to an accepting path for the substring $x[k \dots n-1]$

Map Function: Graph Building

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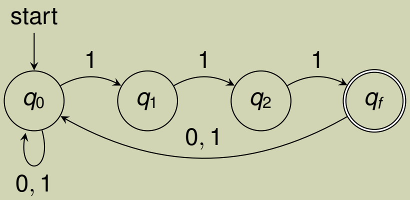
Building the Graph

- Each frontier $\mathcal{S}_k$ becomes a layer of the graph
- Each layer is made of the states $q \in \mathcal{S}_k$
- Each state q is connected to the states of the subsequent layer $q' \in \delta(q, x[k])$

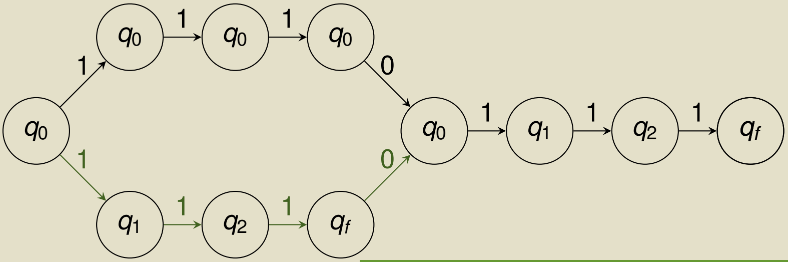
Graph Building

Graph building for $\mathbf{x} = \mathbf{1110111}$

N-DFA for $\mathbf{L} = (\mathbf{0} \mid \mathbf{1})^*\mathbf{111}$



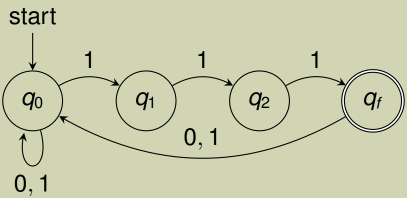
k	$\mathcal{F}_k$	$\mathcal{B}_k$	$\mathcal{S}_k$
0	q_0	q_0	q_0
1	q_0, q_1	q_0, q_1	q_0, q_1
2	q_0, q_1, q_2	q_0, q_2	q_0, q_2
3	q_0, q_1, q_2, q_f	q_0, q_f	q_0, q_f
4	q_0	q_0	q_0
5	q_0, q_1	q_1	q_1
6	q_0, q_1, q_2	q_2	q_2
7	q_0, q_1, q_2, q_f	q_f	q_f



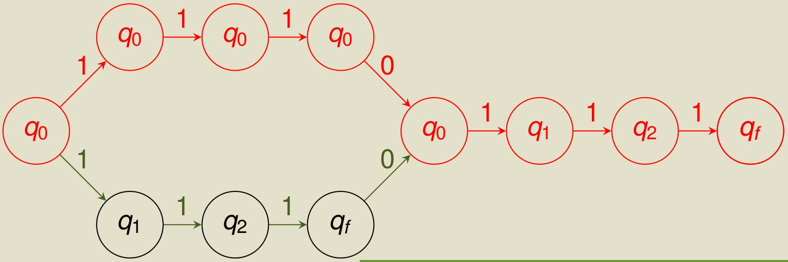
Graph Building

Graph building for x = 1110111

N-DFA for L = (0 | 1)*111



k	$\mathcal{F}_k$	$\mathcal{B}_k$	$\mathcal{S}_k$
0	q_0	q_0	q_0
1	q_0, q_1	q_0, q_1	q_0, q_1
2	q_0, q_1, q_2	q_0, q_2	q_0, q_2
3	q_0, q_1, q_2, q_f	q_0, q_f	q_0, q_f
4	q_0	q_0	q_0
5	q_0, q_1	q_1	q_1
6	q_0, q_1, q_2	q_2	q_2
7	q_0, q_1, q_2, q_f	q_f	q_f



Completing Relaxed Ranking

Unmap

- Performing *unmap* means deriving the string $x \in L$ from an accepting path $\pi \in \mathcal{P}$
- This is straightforward: just follow the transitions of the path and concatenate their characters
- *unmap* is trivially linear in $|x|$

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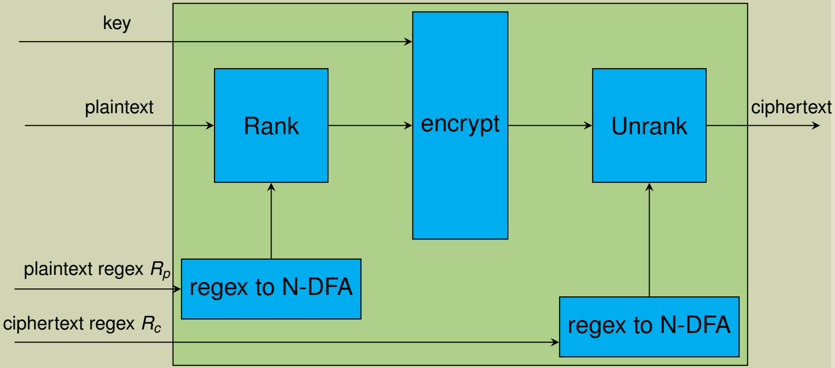
Ranking on Accepting Paths

- In classical ranking, the 2 steps of parsing and computing the rank are done at the same time
- However, the actual ranking is already computed over accepting paths
- Therefore, we can employ the same ranking/unranking procedures, with slight modifications, to rank directly on N-DFA accepting paths

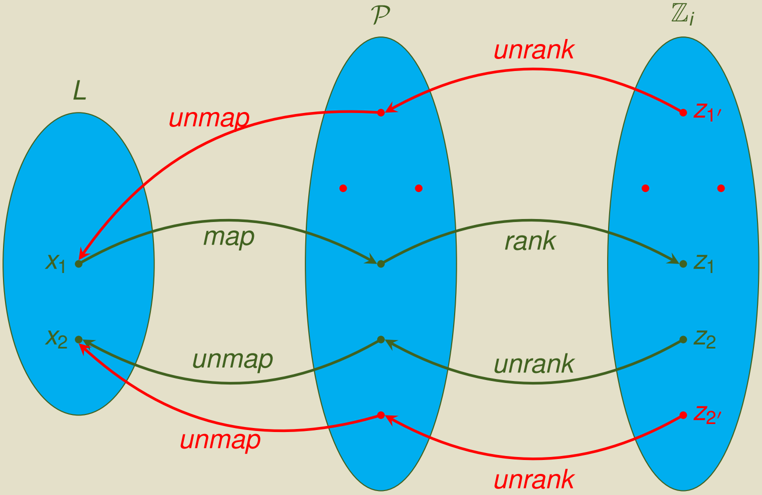
FTE On a Generic Regular Language

Inside the FTE Box



- N-DFA have usually less states than DFA
- Conversion from regex is also faster, since no determinization is involved
- Rank and Unrank are a bit more complex

Invalid Encryptions



Invalid Encryptions

A Further Issue

- Generally, it is unlikely that $\mathbb{Z}_i$ is the domain of the encryption algorithm
- It is expected that the ciphertext domain is a superset of $\mathbb{Z}_i$
- Example: if you use AES-128, the ciphertext domain is $\mathbb{Z}_{2^{128}}$, but likely $i < 2^{128}$
- Therefore, an encryption $c \notin \mathbb{Z}_i$ is an invalid encryption

How to deal with invalid encryptions? We can repeat the encryption until you get a valid ciphertext!

Determine Invalid Encryptions

Ranking Image

- In case of relaxed ranking, the ranking image for a language L is the set $\mathcal{I}(L) = \{z \in \mathbb{Z}_i \mid Rank_L(Unrank_L(z)) = z\}$
- Since *Rank* is injective, $|L| = |\mathcal{I}(L)|$
- Note that these images are likely disjoint intervals of integers in $\mathbb{Z}_i$

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- Since *Rank* is injective, $|L| = |\mathcal{I}(L)|$
- Note that these images are likely disjoint intervals of integers in $\mathbb{Z}_i$

Testing Membership to Ranking Images

- Testing if $z \in \mathbb{Z}_i$ is trivial
- How do we test that the encryption output $z \in \mathbb{Z}_i$ is in a ranking image $\mathcal{I}(L)$?
- Recall that $z \in \mathcal{I}(L) \Leftrightarrow Rank_L(Unrank_L(z)) = z$
- More efficient way: $map_L(Unrank_L(z)) = unrank_L(z)$

Applications: In-Place Encryption

Credit Card Numbers

Suppose we have a database with a table containing a column storing credit card numbers.

- We want to store them encrypted, but without requiring additional memory
- In-place encryption can be achieved with Format Preserving Encryption (FPE)
- Input format $=$ Output format $= [0-9]\{16\}$

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Compression?

- To represent a decimal digits, the DBMS employ at least 4 bits
- For 16 digits $\rightarrow$ 64 bits required
- However, to represent all possible integers of 16 digits, 54 bits are sufficient $\rightarrow$ 7 bytes are enough
- Therefore, we can encrypt using FTE where the input format $= [0-9]16$ and output format $= [0-255]\{7\}$

In-Place Encryption: Performance

Testing Environment

Testing Platform: PostgreSQL 9.1 on Ubuntu 12.04 equipped with Intel Xeon $E3 - 1220$ v3 @ 3.10GHz and 32GB RAM
For performance comparison, 3 different configurations (apart from FPE and FTE) are employed

1. **PSQL**: The default PostgreSQL configuration $\rightarrow$ no encryption is involved
2. **AES**: The data is encrypted using AES-128 in ECB mode
3. **AE**: The data is encrypted with an authenticated encryption scheme (recommended scenario for PostgreSQL)

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Performances

	PSQL	AES	AE	FPE	FTE
Table size(MB)	50	65	112	50	42
Query avg. Time(ms)	74	92	112	125	110

- Reducing size but comparable performances with AE scheme
- Approximately 20% performance overhead against 35% size reduction w.r.t. AES

Deep Packet Inspection

- A deep packet inspector (DPI) filters the network traffic based on the protocol or on the content of the packets
- The protocols allowed by the DPI are called target protocols

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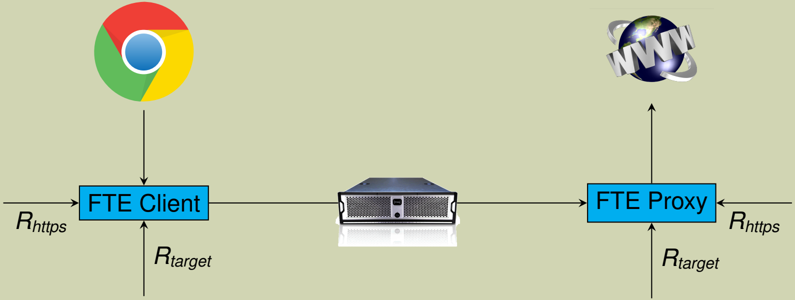
Existing Inspectors

Name	Classifier	Price
appid	regex + port filtering	Free
L7-filter	regex	Free
YAF	regex	Free
Bro	regex + C code to check false positives	Free
Nprobe	regex + flow tracking	300 €
DPI-X	??	10k €

- DPI-X is a proprietary system, but the company did not authorize publication of the name
- It achieves high throughput inspection (1.5$Gbps$), but the classifier is unknown
- However, it has been identified as the device employed by Iran for censorship purposes, thus it is a good device to be tested

Protocol Misclassification Attacks

Attack Scenario



The attack is successful if the DPI is fooled misclassifying the https as the target protocol

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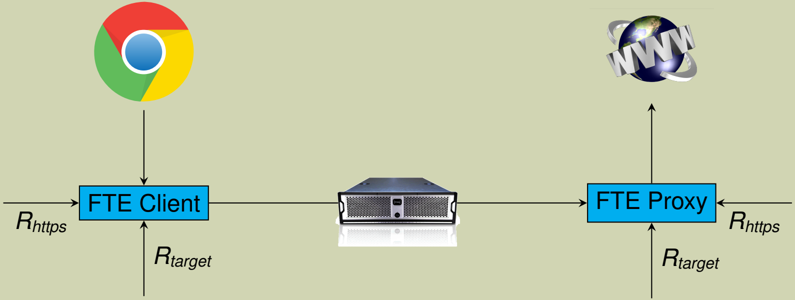

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2 main results:

- Attack succeeds against all the tested DPI (even DPI-X)
- Performance: using FTE tunnel $\approx$ using SSH tunnel

Defeating the Great Firewall of China (GFC)

Chinese government uses DPI for censorship purposes!

Censorship

Connections to Facebook or Youtube are usually dropped. What about Tor?

- It is possible to use Tor, but the GFC is able to actively probe the Tor Onion Proxy (OP) and understand that it is a Tor node
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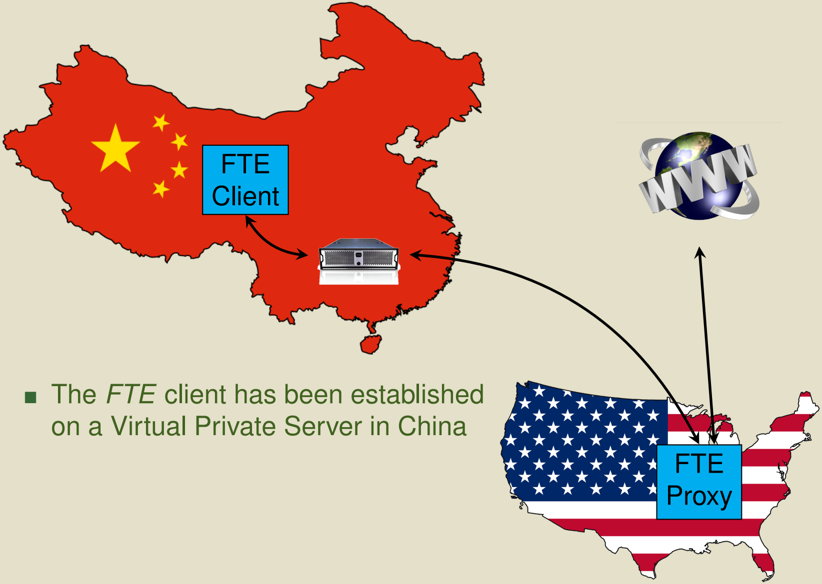
Defeating Censorship

- Defeating website censorships: With FTE, we can map HTTPS packets for Facebook to random HTTP packet
- Defeating Tor censorship: The main issue which triggers actively probing by the GFC is the usage of HTTPS to communicate with Tor OP

$\Downarrow$

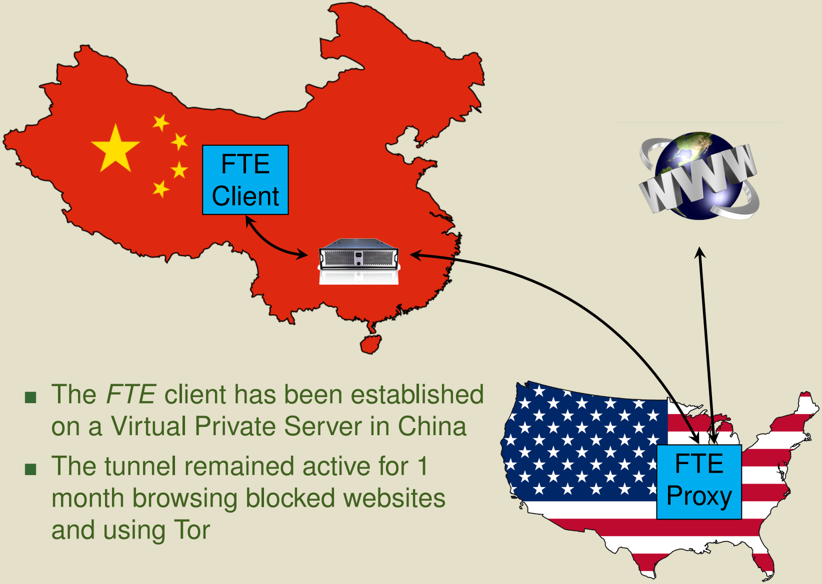
We can use FTE to communicate with the Tor OP!

On the Field Testing



- The *FTE* client has been established on a Virtual Private Server in China

On the Field Testing



- The *FTE* client has been established on a Virtual Private Server in China
- The tunnel remained active for 1 month browsing blocked websites and using Tor

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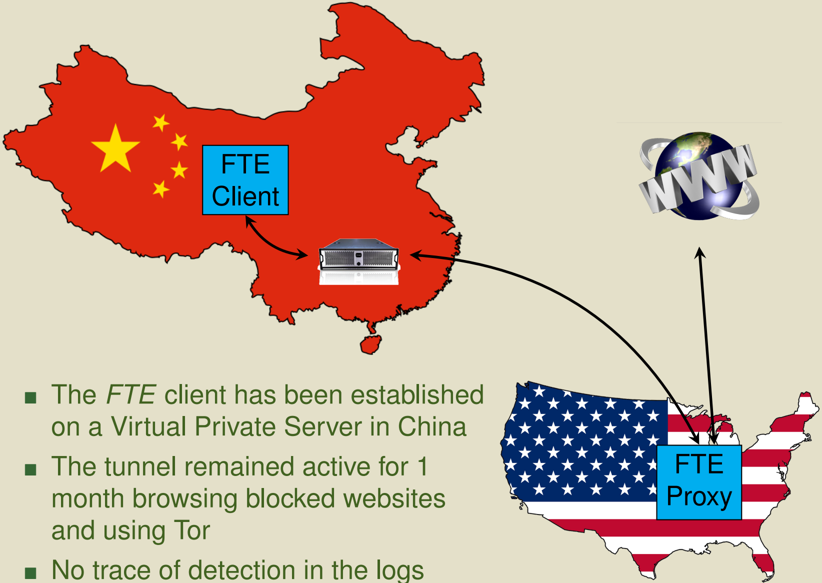

- The *FTE* client has been established on a Virtual Private Server in China
- The tunnel remained active for 1 month browsing blocked websites and using Tor
- No trace of detection in the logs

Countermeasures

Active Probing

- The GFC may actively probe the FTE proxy
- However, differently from the Tor OP case, plain HTTP is used
- Therefore, the GFC should actively probe on HTTP connections too in order to detect FTE $\rightarrow$ high complexity or negligible probability of being caught

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Employing Distribution of HTTP packets

- FTE outputs a random HTTP packet
- Therefore, the distributions of HTTP packets on a tunneled FTE flow or in a normal flow are actually distinguishable
- The DPI can thus set up a distinguisher which is able to spot FTE tunnel
- However, it is still an open problem performing this detection in a scalable way and with negligible overhead for the DPI settings

Further Developments

- Countermeasures based on distinguishing packets distributions is a serious threat to FTE usage against DPI!
- Can’t we extend FTE such that the format is no longer a regex but a probability distribution?

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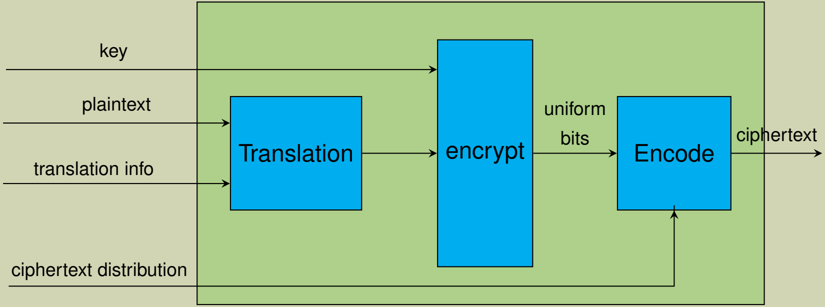
An Improved FTE Box



Main challenge: How to invertibly sample from a generic distribution using uniform bits $\equiv$ Encode box?

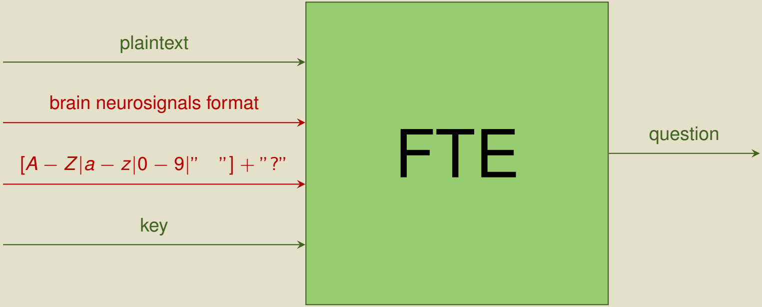
Conclusions

Take Home Messages

- Format Transforming Encryption (FTE) allows to get ciphertexts abiding to a specified format, being able to recover the plaintext in its format
- Format Preserving Encryption (FPE) is a particular case of FTEw, where the ciphertext format is the same as the plaintext one
- FTE is based on ranking/unranking algorithms for regular languages
- To enlarge the set of language which can be efficiently ranked, the relaxed ranking primitive has been introduced
- Relaxed ranking employs the first linear time parsing algorithm for N-DFA
- FPE/FTE are suitable for in-place encryption scenario, achieving a good tradeoff between performance and size reduction
- FTE can successfully be used for protocol misclassification attacks, even against enterprise grade DPI
- A FTE tunnel can be used to circumvent Chinese censorship performed by the Great Firewall of China

Want to try it?: https://github.com/kpdyer/libfte

Questions

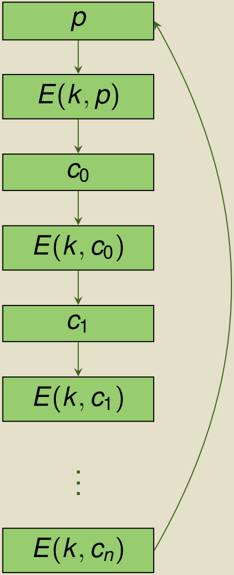


plaintext
brain neurosignals format
$[A-Z|a-z|0-9|"\quad"]+"?"$
key
FTE
question

Invalid Encryptions: Cycle Walking

Workaround for Unrankable Ciphertexts

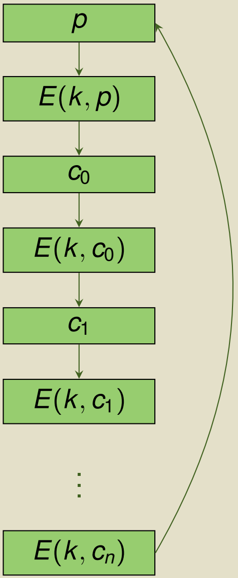
- In a deterministic cipher $c = E(k, p)$, c is constant for the same k, p values
- If c is not a valid ciphertext, we can generate a new ciphertext by computing $c_1 = E(k, c)$
- Iterating this process, we get either back to p or to a valid ciphertext
- This procedure is called cycle walking



Invalid Encryptions: Cycle Walking

Cycle Walking Decryption

- When we decrypt, we perform reverse cycle walking, getting values p_i, to retrieve the original plaintext p
- We retrieve p when we get a $p_i \in \mathcal{I}(L(R_p))$
- Unfortunately, if there is a $p_k \in \mathcal{I}(L(R_p)) \mid k > i$, then $p = p_k \rightarrow$ decryption is wrong!
- Since this situation is not detectable during decryption, we need to modify encryption
- Hence, during encryption, if $c_i \in \mathcal{I}(L(R_p)) \wedge c_i \notin \mathcal{I}(L(R_c)) \rightarrow$ decryption will fail, thus encryption is impossible!



LibFTE Configuration Assistant

```
$ ./configuration-assistant \
>  --input-format "(a|b)*a(a|b){16}" 0 64 \
>  --output-format "[0-9a-f]{16}" 0 16

==== Identifying valid schemes ====
No valid schemes.
ERROR: Input language size greater than
output language size.
$
```

OR

```
$ ./configuration-assistant \
>  --input-format "(a|b)*a(a|b){16}" 0 32 \
>  --output-format "[0-9a-f]{16}" 0 16

==== Identifying valid schemes ====
WARNING: Memory threshold exceeded when
         building DFA for input format
VALID SCHEMES: T-ND, T-NN,
               T-ND-$, T-NN-$

==== Evaluating valid schemes  ====
SCHEME ENCRYPT DECRYPT ... MEMORY
T-ND   0.32ms  0.31ms  ... 77KB
T-NN   0.39ms  0.38ms  ... 79KB
...
$
```